ENUMERATION OF PRIMES.

BY PROF. W. W. JOHNSON, ST. JOHN'S COLLEGE, ANNAPOLIS, MD.

MR. J. W. L. GLAISHER'S enumeration of primes, referred to at page 9, Vol. II of the ANALYST, has been completed, and a "preliminary account" of the results appeared in the Proceedings of the Cambridge Philosophical Society, Vol. III.

In this paper, Mr. Glaisher gives tables of the form described below, for the first three millions of natural numbers comprised in Burckhardt's Tables of Divisors, and for the 7th, 8th and 9th millions which are covered by Dase's Tables.

Calling the hundred numbers between $100n$ and $100(n+1)$ a "century", the tables show, for each of the thirty groups of 100,000 in these six millions, the number of centuries which contain no prime, the number which contain a single prime, two primes, three, and so on; also the total number of primes in each group. These tables are given below, the ten numbers in each column being the numbers of primes in each of the ten groups of the million to which it refers, and the total at the foot, the number of primes in the entire million. The differences are appended to exhibit more strikingly the remarkable inequality which exists in the distribution of primes.

	1st million		2nd million		3rd million	
1st group	9,593		7,216		6,874	
		—1,203		+ 9		—17
2nd "	8,392		7,225		6,857	
		— 379		—144		— 8
3rd "	8,013		7,081		6,849	
		— 150		+ 22		—58
4th "	7,863		7,103		6,791	
		— 185		— 75		—21
5th "	7,678		7,028		6,770	
		— 118		— 55		+39
6th "	7,560		6,973		6,809	
		— 115		+ 42		—44
7th "	7,445		7,015		6,765	
		— 37		— 83		—49
8th "	7,408		6,932		6,716	
		— 85		+ 25		+30
9th "	7,323		6,957		6,746	
		— 99		— 54		—38
10th "	7,224		6,903		6,708	
		— 8		— 29		
Total	78,499		70,433		67,885	

	7th million		8th million		9th million	
1st group	6,397		6,369		6,250	
2nd "	6,402	+ 5	6,306	— 63	6,301	+51
3rd "	6,425	+23	6,348	+ 42	6,283	—18
4th "	6,337	—88	6,299	— 49	6,285	+ 2
5th "	6,347	+10	6,301	+ 2	6,245	—40
6th "	6,402	+55	6,305	+ 4	6,326	+81
7th "	6,338	—64	6,347	+ 42	6,281	—45
8th "	6,375	+37	6,245	—102	6,299	+19
9th "	6,411	+36	6,364	+119	6,220	—79
10th "	6,365	—46	6,274	— 90	6,270	+50
		+ 4		— 24		
Total	63,799		63,158		62,760	

Excepting the first two centuries, which contain 26 and 21 primes respectively, there are no centuries containing more than 17 primes; and excluding the first group of 100,000 numbers, there are no centuries containing more than 15 except two containing 17 primes, one between 700,000 and 800,000 and the other in the third million, and one containing 16 primes in the second million. Two centuries containing 15 primes occur in the eighth million, but none with more than 14 occur in the seventh or ninth. No centuries free from primes occur in the first million, and but one each in the second and third, but the seventh million contains six and the eighth and ninth four each, of such centuries.

Mr. Glaisher defers a comparison of his results with the logarithmic integral formula until after the completion of a sufficiently extended table of values of the integral.

PROPOSITION. BY L. W. MEECH, A. M.—If a, b, c denote three lines drawn from the angles of a plane triangle to the center of the inscribed circle whose radius is r, then will the reciprocals

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{r}$$

be the four sides of a trapezium inscribed in a semicircle; the latter side coinciding with the diameter.